

# Unruh effect and condensate in and out of an accelerated vacuum

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We revisit the Unruh effect to investigate how finite acceleration would affect a scalar condensate. We discuss a negative thermal-like correction associated with acceleration. From the correspondence between thermo-field dynamics and acceleration effects we give an explanation for this negative sign. Using this result and solving the gap equation we show that the condensate should increase with larger acceleration.

## I. INTRODUCTION

Thermal nature inheres in quantum field theory in spacetime with an event horizon and it is characterized by the widely known Hawking-Unruh temperature [1, 2];  $T_H = \kappa/(2\pi)$  for the black-hole case, where  $\kappa = 1/(4M)$  is the surface gravity at the horizon and  $M$  is the black-hole mass. For  $M \sim M_\odot$  (solar mass), this temperature is of the order of  $10^{-8}$  K and it is difficult to detect any direct signal for the Hawking radiation from astrophysical observations. Nevertheless, it is still a fascinating idea to seek for an analogous and more controllable system having an event horizon. In a laboratory setup, the role of surface gravity may be replaced by acceleration leading to the Unruh effect [2–5] (see also Ref. [6] for a recent review). Several interesting ideas have been put forward to test the Hawking-Unruh effect in a laboratory, especially concerning the condensed matter analogue [7], strong field systems [8, 9], lasers [10–16], and heavy ion collisions [17, 18].

The basic premise of the Unruh effect is that an accelerated observer sees the Minkowski vacuum as a thermal (Unruh) bath. Importantly, the Minkowski vacuum is not necessarily empty but sometimes endowed with condensates. In the ground state of Quantum Chromodynamics (QCD), that is commonly called the QCD vacuum, for instance, the chiral condensate makes fermionic (quark) excitations gapped and the gluon condensate arises from the trace anomaly. In the electroweak sector the vacuum accommodates the Higgs condensate and the Higgs phenomena are ubiquitous in condensed matter experiments.

The basic motivation of this work is to understand if a condensate can be modified by a finite acceleration in general. The possible response of a condensate to the Unruh effect is especially intriguing from the point of view of an analogy to thermal environments; in ordinary thermal field theories finite temperature tends to destroy the condensate, while the effect of finite acceleration may be or may not be the same.

Indeed, the original works in the 1980's on the relationship between acceleration and condensates [19, 20] led to a conclusion that acceleration would not have any effect on the condensate. This was motivated by a general theorem in interacting field theories by Unruh and

Weiss [21] on the equality of correlations functions quantized in Rindler and Minkowski spacetimes.

However, a number of recent works have found strikingly different results. The investigation of Ref. [22] (see also Ref. [23]) on the quark-antiquark scalar condensate (that is, the chiral condensate) in a Nambu–Jona-Lasinio model concluded that the condensate should decrease as a function of increasing acceleration. A similar conclusion was reached in Ref. [24] for a quark-quark (i.e. diquark) condensate that realizes in color-superconducting phases. In Refs. [25–27] a real scalar field theory was investigated in an accelerating frame, with the main result being that a scalar condensate would decrease at finite acceleration. Moreover, holographic models of QCD [28–30] indicate that acceleration acts to weaken the interaction between quarks and antiquarks in hadrons leading to a deconfinement transition. In summary, according to these preceding works, phases with a finite scalar condensate (and also confining effects) would eventually be destroyed at some high acceleration, in a similar way as it occurs at high temperature.

The interpretation of the acceleration effect on condensates is far less clear than that of the temperature effect, a part of which should be attributed to different physical setups. This makes it imperative to reconsider the thermal-like effects in accelerated systems. Thus, in this work we revisit a real scalar field theory in Rindler spacetime. We explicitly compute the Wightman two-point function in the Rindler (accelerated) vacuum. The most subtle part is the treatment of the ultraviolet (UV) divergence in the Rindler and the Minkowski vacua. To make our assumption clear, we discuss the role played by the observer; the observer knows the energy dispersion relation and defines the particle there. Then, two-point functions involving only field operators but not the dispersion relation are insensitive to which of the Rindler and the Minkowski vacua is chosen for the field quantization. Besides, our observer would not reorganize the vacuum structure. This means that we should treat the UV divergence in the same way as the finite-temperature field theory, so that we can focus only on a finite correction induced by acceleration. Interestingly, we will see that this acceleration-induced correction has a sign opposite to what is expected as a thermal correction. We develop an analogy to the formalism of thermo-field dy-

namics (TFD) to clarify the the origin of this opposite sign.

This opposite sign reverses the role of thermal-like effects and brings an exotic possibility that a larger condensate could be favored with increasing acceleration. This is the case when the condensate is observed in the co-accelerated frame. Utilizing the one-loop mean-field approximation, we will solve the equation of motion and calculate the condensate numerically and analytically. We employ a boundary condition that ensures that we can smoothly reach a Minkowski-vacuum limit when the acceleration is turned off. Our solution exhibits divergent behavior of the condensate as a function of the acceleration. We might well call such a property of accelerated matter “acceleration catalysis” in analogy to the magnetic catalysis [31].

The nature of condensates on non-trivial spacetime manifolds [32–34] and in non-inertial frames, i.e. accelerating or rotating frames [35], is an interesting topic in general, and our results should be valuable in that perspective. Especially interesting is the case with the Schwarzschild metric [36] which takes the form of Rindler spacetime near the horizon. Hence, based on our finding we will give a brief remark about a possible implication to the condensation phenomena in the vicinity of the black hole.

In order to appreciate the difference from finite temperature physics, we carefully layout the Rindler spacetime formalism and the Bogolyubov transformation in a scalar field theory in Sec. III. Readers familiar with this description may skip this part and proceed directly to Sec. IV where we discuss the basic expressions about field correlations and number operators with acceleration. Also we discuss the correspondence between the accelerated vacuum and the thermal vacuum. Such a careful comparison provides us with a key to the phenomena of acceleration catalysis which is addressed in Sec. V. We make our conclusions in Sec. VI. We give an explicit check of the equality between the Wightman two-point functions in the Rindler and the Minkowski vacua in Appendix.

## II. INEQUIVALENT VACUA AND OBSERVERS

We would stress the importance to sort out the definitions of the *vacua* (or *states*) and the *observers* (or *operators*) first. Let us start our discussions with an analogous and more intuitive example of particle production under an external electric field, i.e. the phenomenon called the Schwinger mechanism [37]. The problem with an electric field is essentially dynamical in a sense that the background gauge fields should be time dependent. It is convenient to introduce quantities and operators in the infinitely past (and future) state that are referred to with a subscript “in” (and “out” respectively).

It is a well-known result that the in-vacuum is not really a vacuum if seen by an observer sitting in the out-vacuum, which is explicitly expressed in a form of the in-

state expectation value of the out-state number operator. For example, if the electric field is applied for a time  $\sim t$  along the positive  $z$ -direction, the production of charged bosonic particles results in a distribution as follows [38]:

$$\langle \text{in} | \hat{a}_{\text{out}}^\dagger(\mathbf{k}) \hat{a}_{\text{out}}(\mathbf{k}) | \text{in} \rangle \sim \exp \left[ -\frac{\pi(\mathbf{k}_\perp^2 + m^2)t}{4} \left( \frac{1}{k^z - eEt} + \frac{1}{k^z} \right) \right] \quad (1)$$

for  $0 \leq k^z \leq eEt$ . This non-zero result appears from the Bogolyubov coefficients between  $\hat{a}_{\text{in}}$ ,  $\hat{a}_{\text{in}}^\dagger$  and  $\hat{a}_{\text{out}}$ ,  $\hat{a}_{\text{out}}^\dagger$ .

In Eq. (1) the in-vacuum  $|\text{in}\rangle$  is probed by an out-operator. In other words, the observer *defines* the operator we should put in the expectation value. To understand this machinery more, it would be instructive to recall how the number operator can be written in terms of field operators. As derived in Ref. [39], we can show:

$$\hat{a}_{\text{out}}^\dagger(\mathbf{k}) \hat{a}_{\text{out}}(\mathbf{k}) = \frac{1}{2\varepsilon_{\text{out}}(\mathbf{k})} \lim_{t_1=t_2 \rightarrow \infty} \times [\partial_{t_1} + i\varepsilon_{\text{out}}(\mathbf{k})][\partial_{t_2} + i\varepsilon_{\text{out}}(\mathbf{k})] \hat{\phi}^\dagger(t_1, \mathbf{k}) \hat{\phi}(t_2, \mathbf{k}), \quad (2)$$

where  $\varepsilon_{\text{out}}(\mathbf{k})$  is the energy dispersion relation in the out-state which generally depends on when and where the particle is observed. Importantly, as we will explicitly confirm later, the field operators,  $\hat{\phi}$  and  $\hat{\phi}^\dagger$ , are not sensitive to the detection procedures. A more familiar and general example of the relevance of the out-observer through the energy dispersion relation can be also found in the famous LSZ (Lehmann-Symanzik-Zimmermann) reduction formula.

Now, we shall turn to the consideration about the Unruh effect. It is crucial to make the observer’s role clear in order to clarify the physical interpretation of the Unruh effect. We will see later that the non-accelerated vacuum expectation value of the number operator in the accelerated vacuum,  $\langle M | \hat{a}_R^\dagger \hat{a}_R | M \rangle$ , has a thermal spectrum whose temperature is characterized by the acceleration. Then, one might be tempted to consider that this non-accelerated vacuum could be a thermal bath giving rise to thermal-like corrections. Such an argument would cause confusion if applied too naïvely, and the fact is that there is no such thermal-like correction as long as an operator is written in terms of  $\hat{\phi}$  and  $\hat{\phi}^\dagger$  only and not with the energy dispersion relation inherent in the observer.

Nevertheless, even for operators not involving the energy dispersion relation, it is still a non-trivial question how the operator expectation value may change with different vacua; the non-accelerated vacuum  $|M\rangle$  and the accelerated one  $|R\rangle$ . This is a question that we elucidate in the present work. In particular, we are interested in a scalar condensate affected by the acceleration. In summary, we will take a close look at the question:

$$\langle M | \hat{\phi} | M \rangle \stackrel{?}{=} \langle R | \hat{\phi} | R \rangle \quad (3)$$

and think about underlying physical interpretations in analogy to thermal field theory in what follows below.

### III. UNRUH EFFECT IN A SCALAR FIELD THEORY

This is an overview section and we summarize our notations and choice of the coordinates, i.e. those in Rindler spacetime. These preliminary setups are important for the later analysis on the spontaneous symmetry breaking in Sec. V.

#### A. Scalar field in Rindler spacetime

The Minkowski metric is given as  $ds^2 = dt^2 - d\mathbf{x}_\perp^2 - dz^2$  in our convention where  $\mathbf{x}_\perp = (x, y)$ . We perform a change of coordinate variables from  $t$  and  $z$  to  $\rho$  and  $\eta$ , which defines the Rindler coordinates as follows:

$$z = \rho \cosh \eta, \quad t = \rho \sinh \eta \quad (4)$$

with the metric in a form of

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - d\mathbf{x}_\perp^2. \quad (5)$$

These new coordinates,  $\rho$  and  $\eta$ , cover only a part of the Minkowski spacetime as long as  $\rho$  is non-negative and  $\eta$  is real. Because the region of  $z > |t|$  is spanned then, we call  $\rho$  and  $\eta$  the *right-wedge* Rindler coordinates. We can also introduce another coordinates,  $\bar{\rho}$  and  $\bar{\eta}$ , to define the *left-wedge* Rindler coordinates in a similar fashion. In this work we will focus on the right-wedge Rindler coordinates only; we can setup an accelerated particle trajectory within this  $z > 0$  region without loss of generality.

Using the notion of the proper time  $\tau$  and the velocity four-vector  $u^\mu = dx^\mu/d\tau$ , we can define the acceleration four-vector as  $a^\mu = du^\mu/d\tau$ . Then, the proper acceleration  $\alpha$  is given by

$$\alpha^2 = -a_\mu a^\mu. \quad (6)$$

A trajectory of a point particle with a proper acceleration  $\alpha$  in terms of the Minkowski coordinates can be parametrized as

$$z(\tau) = \frac{1}{\alpha} \cosh(\alpha\tau), \quad t(\tau) = \frac{1}{\alpha} \sinh(\alpha\tau). \quad (7)$$

Therefore, in terms of the right-wedge Rindler coordinates, this trajectory corresponds to  $\eta = \alpha\tau$  with a fixed value of  $\rho = 1/\alpha$ . Thus, we should stress here that  $\rho$  and  $\eta$  have dual roles as coordinates and parameters characterizing an accelerated trajectory. As sketched in Fig. 1, the constant- $\rho$  trajectories move away from the light-cone, and their shape straightens, as  $\rho$  increases. This clearly means that a larger  $\rho$  represents a smaller acceleration, which is consistent with the identification of  $\rho = 1/\alpha$ .

The action for a real scalar field theory in a general coordinate system (apart from the curvature) [40] reads:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (8)$$

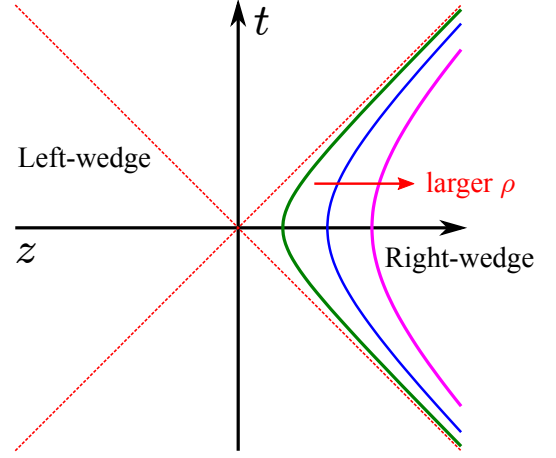


FIG. 1. Schematic picture of the Rindler coordinates. In the right-wedge Rindler coordinates several curves with different values of  $\rho$  are shown; a larger  $\rho$  makes the constant- $\rho$  trajectory straightened corresponding to a less acceleration.

where  $V(\phi)$  is a potential term.

Let us now setup two distinct observers; a Minkowski (non-accelerated) observer and a Rindler (accelerated) observer. The Minkowski and the Rindler observers quantize the fields based on the energy dispersion relation in Minkowski and Rindler space, respectively. We will indicate observables quantized in this way by denoting  $\hat{\mathcal{O}}_M$  and  $\hat{\mathcal{O}}_R$  for the Minkowski and the Rindler observers. As we already mentioned above, this distinction is irrelevant for observables in terms of  $\hat{\phi}$  and  $\hat{\phi}^\dagger$  only.

We briefly see how the Unruh effect is derived. With the Minkowski coordinates the quantum field is expanded in terms of a complete set of basis functions (denoted by  $f$ 's) with the creation and annihilation operators as

$$\hat{\phi} = \int d^3k \left[ \hat{a}_M(\mathbf{k}) f(\mathbf{k}, x) + \hat{a}_M^\dagger(\mathbf{k}) f^*(\mathbf{k}, x) \right]. \quad (9)$$

The choice of the complete set is arbitrary and it would be the most convenient one to take the plane waves as the basis functions as

$$f(\mathbf{k}, x) = \frac{1}{(2\pi)^{3/2} (2k_0)^{1/2}} e^{i\mathbf{k} \cdot \mathbf{x} - ik_0 t}. \quad (10)$$

We thus define the Minkowski vacuum  $|M\rangle$  as a solution of  $\hat{a}_M(\mathbf{k})|M\rangle = 0$ .

With the Rindler metric (5) we should replace  $f(\mathbf{k}, x)$  with a counterpart of the plane wave in terms of the right-wedge Rindler coordinates (see, e.g. Refs. [6, 25, 41] for technical details) given by

$$f_R(\mathbf{k}_\perp, \omega, x) = \frac{\sqrt{1 - e^{-2\pi\omega}}}{[2(2\pi)^4]^{1/2}} K\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp - i\omega\eta}, \quad (11)$$

where we introduced  $\kappa \equiv \sqrt{\mathbf{k}_\perp^2 + m^2}$ . We note that  $\omega$  is a dimensionless conjugate of  $\eta$ . From the trajectory,

$\eta = \alpha\tau$ , we should understand that  $\alpha\omega$  corresponds to a Rindler energy. The  $\rho$  dependence appears through a special function defined as

$$K(\omega, \alpha, \beta) \equiv \int_0^\infty \frac{ds}{s} s^{i\omega} e^{-is\alpha + i\beta/s}. \quad (12)$$

We should consider this function in the physical region of  $\omega > 0$  where we can check the following:

$$K(\omega, \alpha, \alpha) = 2e^{\pi\omega/2} K_{i\omega}(2\alpha). \quad (13)$$

Here,  $K_{i\omega}(x)$  represents the modified Bessel function of imaginary order. We can then expand the quantum field for the Rindler observer as

$$\hat{\phi} = \int_0^\infty d\omega \int d^2k_\perp \left[ \hat{a}_R(\mathbf{k}_\perp, \omega) f_R(\mathbf{k}_\perp, \omega, x) + \hat{a}_R^\dagger(\mathbf{k}_\perp, \omega) f_R^*(\mathbf{k}_\perp, \omega, x) \right]. \quad (14)$$

In the same way as for Minkowski spacetime, we can define the vacuum for the Rindler observer by solving  $\hat{a}_R(\mathbf{k}_\perp, \omega)|R\rangle = 0$ . It is also possible to perform an expansion of the field in the left-wedge to construct another vacuum from  $\hat{a}_L(\mathbf{k}_\perp, \omega)|L\rangle = 0$  if necessary.

### B. Bogolyubov transformation and the Unruh temperature

We establish the relation between  $\hat{a}_M(\mathbf{k})$  and  $\hat{a}_{R/L}(\mathbf{k}_\perp, \omega)$ , which is formulated conveniently in terms of the Bogolyubov coefficients. Even though the transformation (4) is just a change of variables, the existence of a causal horizon at  $t = \pm z$  for the accelerated observer introduces a non-trivial structure through the Bogolyubov transformation and this is at the heart of the Unruh effect.

Following Ref. [41] we first define a light-cone annihilation operator as  $\sqrt{k^+} \hat{a}_1(\mathbf{k}_\perp, k^+) = \sqrt{k_0} \hat{a}_M(\mathbf{k})$  with  $k^\pm \equiv (k^z \pm k^0)/\sqrt{2}$ . Then, we find another operator by a variable change from  $k^+$  to  $\omega$ , i.e.

$$\hat{a}_2(\mathbf{k}_\perp, \omega) = \int_0^\infty \frac{dk^+}{(2\pi k^+)^{1/2}} \hat{a}_1(\mathbf{k}_\perp, k^+) e^{i\omega \log[(k^+ \sqrt{2})/\kappa]}. \quad (15)$$

For  $\omega > 0$  these new operators  $\hat{a}_2(\mathbf{k}_\perp, \pm\omega)$  are related to the Rindler operators  $\hat{a}_{R/L}(\mathbf{k}_\perp, \omega)$  through a linear transformation expressed as

$$\begin{pmatrix} \hat{a}_R(\mathbf{k}_\perp, \omega) \\ \hat{a}_L(\mathbf{k}_\perp, \omega) \\ \hat{a}_R^\dagger(-\mathbf{k}_\perp, \omega) \\ \hat{a}_L^\dagger(-\mathbf{k}_\perp, \omega) \end{pmatrix} = \begin{pmatrix} \alpha_\omega & 0 & 0 & \beta_\omega \\ 0 & \alpha_\omega & \beta_\omega & 0 \\ 0 & \beta_\omega & \alpha_\omega & 0 \\ \beta_\omega & 0 & 0 & \alpha_\omega \end{pmatrix} \begin{pmatrix} \hat{a}_2(\mathbf{k}_\perp, \omega) \\ \hat{a}_2(\mathbf{k}_\perp, -\omega) \\ \hat{a}_2^\dagger(-\mathbf{k}_\perp, \omega) \\ \hat{a}_2^\dagger(-\mathbf{k}_\perp, -\omega) \end{pmatrix} \quad (16)$$

with the Bogolyubov coefficients given by

$$\alpha_\omega = \frac{1}{\sqrt{1 - e^{-2\pi\omega}}}, \quad \beta_\omega = \frac{e^{-\pi\omega}}{\sqrt{1 - e^{-2\pi\omega}}}. \quad (17)$$

Using the above Bogolyubov transformation and the operator definition (15), we can readily see that the Minkowski vacuum expectation value of the Rindler number operator is non-zero, namely,

$$\begin{aligned} \langle M | \hat{a}_R^\dagger(\mathbf{k}_\perp, \omega) \hat{a}_R(\mathbf{k}'_\perp, \omega') | M \rangle \\ = \beta_\omega \beta_{\omega'} \delta^{(2)}(\mathbf{k}_\perp - \mathbf{k}'_\perp) \int_0^\infty \frac{dk^+}{2\pi k^+} e^{-i(\omega - \omega') \log[(k^+ \sqrt{2})/\kappa]} \\ = \frac{\delta(\omega - \omega') \delta^{(2)}(\mathbf{k}_\perp - \mathbf{k}'_\perp)}{e^{\omega/T_U} - 1}, \end{aligned} \quad (18)$$

which demonstrates the Unruh effect [2], where  $T_U = 1/(2\pi)$  is the dimensionless Unruh temperature. (In the physical unit,  $\alpha\omega$  is a Rindler energy, and so  $\alpha/(2\pi)$  is to be identified as the temperature.) One may be tempted to interpret  $|M\rangle$  as a thermal bath for operators quantized on the Rindler vacuum. Such an argument for the thermal interpretation could be found in some literature, but it is sometimes concluded in a rather misleading manner. We will elucidate this point in more details in the next section.

## IV. CALCULATING THERMAL-LIKE CORRECTIONS

In this section we shall consider the effect of quantum fluctuations in and out of an accelerated vacuum. For this purpose we take an example of two-point Wightman function that is necessary for the evaluation of the scalar condensate. In particular, the coincidence limit of the two-point functions,  $\langle R | \hat{\phi}^2 | R \rangle$  and  $\langle M | \hat{\phi}^2 | M \rangle$ , will introduce a “temperature” dependent mass term in the effective potential [42].

### A. Insensitivity to the observer

Although  $\langle M | \hat{a}_R^\dagger \hat{a}_R | M \rangle \neq \langle M | \hat{a}_M^\dagger \hat{a}_M | M \rangle = 0$  and  $\langle R | \hat{a}_M^\dagger \hat{a}_M | R \rangle \neq \langle R | \hat{a}_R^\dagger \hat{a}_R | R \rangle = 0$ , the Bogolyubov coefficients guarantee that  $\hat{\phi}$  represents the same quantum field. In fact, for  $\mathcal{O}(\hat{\phi})$  not having the energy dispersion relations, it has been addressed based on the functional integration in the literature [21] that  $\langle M | \mathcal{O}(\hat{\phi}) | M \rangle$  does not depend on the choice of the observer who quantizes  $\hat{\phi}$ . Moreover, Refs. [19, 20] utilized the Schrödinger functional formalism with an explicit point-splitting regularization to prove that  $\langle M | \mathcal{O}(\hat{\phi}) | M \rangle$  should be independent of the observer.

This is all so by construction, and nevertheless, operators for different observers (i.e. quantized in different vacua) sometimes cause confusions. Thus, it would be useful to take a glance at how the insensitivity follows explicitly from a proper combination of the vacuum definitions and the Bogolyubov coefficients. The calculation to confirm the insensitivity of both  $\langle M | \mathcal{O}(\hat{\phi}) | M \rangle$  and



$\langle R|\mathcal{O}(\hat{\phi})|R\rangle$  for different observers is tedious but straightforward. This is a two-step procedure to use the Bogolyubov relations (16) and the transformation of plane waves to

$$e^{i(k^+z^- + k^-z^+)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega\eta} e^{i\omega \log(k^+ \sqrt{2}/\kappa)} K\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right), \quad (19)$$

and vice versa. We give more detailed and complete calculations in the Appendix A.

### B. Regularization prescription

We will explicitly evaluate the coincidence limit  $x' \rightarrow x$  of two-point Wightman functions for  $m = 0$ . For concrete calculations we could use the point-splitting regularization. For the  $m = 0$  case, then, we have:

$$\langle M|\hat{\phi}(x)\hat{\phi}(x')|M\rangle = -\frac{1}{4\pi} \frac{1}{(t-t'-i\epsilon)^2 - |\mathbf{x}_{\perp} - \mathbf{x}'_{\perp}|^2 - |z-z'|^2}. \quad (20)$$

For the Rindler vacuum also the Wightman function  $\langle R|\hat{\phi}(x)\hat{\phi}(x')|R\rangle$  has a singular term in the coincident limit just given by Eq. (20) (see also Refs. [19, 20]) as well as a finite deviation. Such a finite extra term should be well-defined irrespective to the ultraviolet regularization. We can actually find:

$$\begin{aligned} \langle R|\hat{\phi}^2|R\rangle_{\text{reg}} &\equiv \lim_{x' \rightarrow x} (\langle R|\hat{\phi}(x)\hat{\phi}(x')|R\rangle - \langle M|\hat{\phi}(x)\hat{\phi}(x')|M\rangle) \\ &= -\frac{1}{4\pi^4} \int_0^{\infty} d\omega e^{-\pi\omega} \int d^2k_{\perp} K_{i\omega}^2(k_{\perp}\rho) \\ &= -\frac{1}{2\pi^2\rho^2} \int_0^{\infty} \frac{\omega d\omega}{e^{2\pi\omega} - 1} = -\frac{1}{48\pi^2\rho^2} \end{aligned} \quad (21)$$

for the  $m = 0$  case. Recalling that the trajectory of a constant proper acceleration  $\alpha$  is defined as  $\rho = 1/\alpha$  and defining the local Unruh temperature as  $T_{\text{loc}} = \alpha/(2\pi)$ , we can interpret this result as a thermal-like correction by

$$\langle R|\hat{\phi}^2|R\rangle_{\text{reg}} = -\frac{T_{\text{loc}}^2}{12}. \quad (22)$$

It should be noted that this expression (22) has a sign opposite to the ordinary thermal correction if  $|R\rangle$  is given an interpretation as a thermal bath [19, 20, 25, 43, 44].

Before closing this subsection, we make a remark that in the canonical quantization it is a conventional procedure to take the normal ordering to discard zero-point oscillation energies, that is,

$$:\hat{\mathcal{O}}_{R/M}:\equiv \hat{\mathcal{O}}_{R/M} - \langle R/M|\hat{\mathcal{O}}_{R/M}|R/M\rangle, \quad (23)$$

where the second contribution represents the discarded divergent piece in the normal ordering in terms of  $\hat{a}_R$

and  $\hat{a}_R^{\dagger}$  and in terms of  $\hat{a}_M$  and  $\hat{a}_M^{\dagger}$ , respectively. In this case, even when  $\mathcal{O}_{R/M}$  has no explicit dependence on the energy dispersion relation, the expectation value may change according to the observer through  $\langle R/M|\hat{\mathcal{O}}_{R/M}|R/M\rangle$ . It is obvious that the subtraction (21) coincides with the normal ordering for the Minkowski observer.

Because we have no complete description of the zero-point oscillation but dropping it with some working prescriptions in quantum field theory, we should choose a reference point where we make a subtraction of the divergent term as in Eq. (21). As emphasized in Sec. II our assumption about the observer is that the observer defines the energy dispersion relation that is needed for switching to the particle picture. Hence, in our prescription, the observer *does not reorganize* the vacuum, and so an offset of the energy level should be intact. This assumption thus prescribes us not to include this difference between  $\langle R|\hat{\mathcal{O}}_R|R\rangle$  and  $\langle M|\hat{\mathcal{O}}_M|M\rangle$  in our computation. In a sense our treatment of the UV singularity is analogous to that in finite-temperature field theory; once divergences are subtracted at  $T = 0$ , no additional divergence appears from  $T \neq 0$  corrections. More specifically, for a thermal state  $|\beta\rangle$  with temperature  $T$ , for a free massless scalar theory, we know:

$$\begin{aligned} \langle \beta|\hat{\phi}^2|\beta\rangle_{\text{reg}} &\equiv \lim_{x' \rightarrow x} (\langle \beta|\hat{\phi}(x)\hat{\phi}(x')|\beta\rangle - \langle 0|\hat{\phi}(x)\hat{\phi}(x')|0\rangle) \\ &= \frac{T^2}{12}, \end{aligned} \quad (24)$$

which is quite suggestive as compared to Eq. (22). We shall pursue for this analogy to finite- $T$  field theory more in the next subsection.

### C. Analogue to thermo-field dynamics

We see a clear correspondence from expectation values of the number operator in Rindler spacetime and in thermal environments. Indeed, a striking similarity is found, which takes the form of

$$\langle \beta|\hat{a}_0^{\dagger}(\mathbf{k})\hat{a}_0(\mathbf{k}')|\beta\rangle = \frac{\delta^{(3)}(\mathbf{k} - \mathbf{k}')}{e^{\sqrt{\mathbf{k}^2 + m^2}/T} - 1}, \quad (25)$$

where  $\hat{a}_0$  and  $\hat{a}_0^{\dagger}$  are the annihilation and creation operators in Minkowski spacetime.

The comparison with thermo-field dynamics (TFD) will provide us with an intuitive understanding of the results (21) and (22). In TFD one deals with the thermal vacuum  $|\beta\rangle$  which is represented by the so-called non-tilde  $|0\rangle$  and tilde  $|\tilde{0}\rangle$  vacua [45] and so  $|\beta\rangle$  is excited relative to  $|0\rangle$ .

It is important to recognize that  $|M\rangle$  should be “thermal” in terms of  $|R\rangle$  and  $|L\rangle$  (right-wedge and left-wedge Rindler vacua that never talk to each other), so that  $|M\rangle$  in Rindler spacetime is analogous to  $|\beta\rangle$  in TFD and  $|R\rangle$  and  $|L\rangle$  should correspond to  $|0\rangle$  and  $|\tilde{0}\rangle$ . We summarize

TFD $\Leftrightarrow$ Rindler Spacetime	
$ \beta\rangle$	$ M\rangle$
$ 0\rangle$	$ R\rangle$
$ \tilde{0}\rangle$	$ L\rangle$

TABLE I. Correspondence between different vacua in TFD and in Rindler spacetime.

the relation among them in Table I. Confusions sometimes arise from misidentification of  $|R\rangle$  as a thermal mixed state, but the fact is opposite. This point is important to understand the meaning of the negative sign in Eq. (22).

In TFD, the observer quantizes operators in a zero temperature vacuum and can measure  $\langle 0|\hat{\mathcal{O}}|0\rangle$  to take it as a “reference” value. Now, let us suppose that we have box representing a piece of material heated to non-zero  $T$ . This box is in a thermal state  $|\beta\rangle$  and the same observer should find:

$$\langle \beta|\hat{\mathcal{O}}|\beta\rangle > \langle 0|\hat{\mathcal{O}}|0\rangle \quad (26)$$

for a positive definite operator  $\hat{\mathcal{O}}$  such as  $\hat{\mathcal{O}} = \hat{\phi}^2$ . The left-hand side in the above receives a thermal correction  $\sim T^2$ , which is a finite correction associated with the temperature effect.

Now, let us imagine a similar experiment where the vacuum state in the box is not heated but accelerated. In the case with acceleration, according to Table I, we should anticipate:

$$\langle M|\hat{\mathcal{O}}|M\rangle > \langle R|\hat{\mathcal{O}}|R\rangle \quad (27)$$

as a counterpart of the relation (26). We should take the left-hand side in the above as our reference point before acceleration, so that a finite correction associated with the acceleration effect is naturally negative. In other words, we can say that the accelerated vacuum is *less excited* as compared to the non-accelerated (Minkowski) vacuum.

## V. SPONTANEOUS SYMMETRY BREAKING

We are considering a real scalar field theory with  $Z_2$  symmetry, which is assumed to be spontaneously broken in the Minkowski vacuum through the potential of the following form:

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4. \quad (28)$$

On the tree level, the state with a minimal energy favors a finite condensate given by

$$\langle M|\hat{\phi}|M\rangle = \sqrt{\frac{\mu^2}{\lambda}}. \quad (29)$$

The question we are addressing in this section is the following; let us consider a box of, say, a superconducting material with a non-zero homogeneous condensate such as in Eq. (29). Then, we accelerate this box and adiabatically change  $|M\rangle$  to  $|R\rangle$  to investigate whether the condensate may increase or decrease with acceleration.

### A. One-loop equation of motion

We here introduce a notation;  $\bar{\phi} \equiv \langle R|\hat{\phi}|R\rangle$  and we can determine  $\bar{\phi}$  by the condition to extremize the effective action. The one-loop calculation in the mean-field approximation leads to the follow equation of motion:

$$(\square - 3\lambda\langle\hat{\phi}^2\rangle_{\text{reg}})\bar{\phi} - V'(\bar{\phi}) = 0. \quad (30)$$

This is a non-linear equation for  $\bar{\phi}$  involving quantum fluctuations encoded in  $\langle\hat{\phi}^2\rangle_{\text{reg}} = \langle R|\hat{\phi}^2|R\rangle - \langle M|\hat{\phi}^2|M\rangle$ . We should note that our reference point is the Minkowski vacuum and so  $\langle\hat{\phi}^2\rangle_{\text{reg}} < 0$ .

Our goal is to find  $\bar{\phi}$  as a function of  $\rho$ , where  $\rho$  is the Rindler coordinate as introduced before. In contrast to finite temperature physics where the effective *potential* is sufficient to fix a condensate, in the acceleration case the Hamiltonian depends on acceleration through the coordinate  $\rho$  and so it is indispensable to keep the derivatives to find  $\bar{\phi}$  in the accelerated vacuum.

We can adopt the quantum fluctuation  $\langle\hat{\phi}^2\rangle_{\text{reg}}$  from Sec. III and approximately use Eq. (21). The present theory is not a massless one, but this massless approximation would simplify the analysis significantly not losing qualitative features. Assuming a non-trivial  $\rho$  dependence in the condensate, the problem boils down to solving the following equation:

$$\frac{d^2\bar{\phi}}{d\rho^2} + \frac{1}{\rho}\frac{d\bar{\phi}}{d\rho} - \nu^2\frac{\bar{\phi}}{\rho^2} = V'(\bar{\phi}), \quad \nu^2 = -\frac{\lambda}{16\pi^2}. \quad (31)$$

Solving Eq. (31) allows for a particle-like interpretation;  $\bar{\phi}$  is to be interpreted as “position” of a particle and  $\rho$  as “time”. Then, this identification enables us to rewrite Eq. (31) in the “energy” form:

$$\frac{d}{d\rho}\left[\frac{1}{2}\left(\frac{d\bar{\phi}}{d\rho}\right)^2 - V(\bar{\phi})\right] = -\frac{1}{\rho}\left(\frac{d\bar{\phi}}{d\rho}\right)^2 + \nu^2\frac{d\bar{\phi}}{d\rho}\frac{\bar{\phi}}{\rho^2}, \quad (32)$$

which gives us an interpretation that a particle is moving in a potential  $-V(\bar{\phi})$ . It is crucial to point out that both terms in the right-hand side are negative (if  $d\bar{\phi}/d\rho > 0$ ), leading to an energy loss as a function of time.

This type of analysis is typical for the calculation of false vacuum decay [46] when a potential energy has several inequivalent minima. One is then interested in finding an “instanton” solution that represents a trajectory from one to the other extrema of the potential  $-V(\bar{\phi})$ . The solution of our current interest should satisfy a boundary condition:

$$\bar{\phi}(\rho \rightarrow \infty) = \sqrt{\frac{\mu^2}{\lambda}}, \quad (33)$$

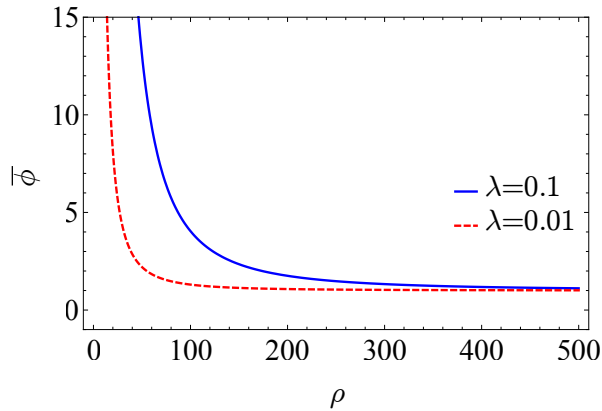


FIG. 2. One-loop corrected background field for  $\lambda = 0.1$  and  $\lambda = 0.01$  as a function of  $\rho = 1/\alpha$  where  $\alpha$  is the proper acceleration. The axes are given in dimensionless units, where  $\bar{\phi}$  and  $\rho$  are scaled by  $\sqrt{\mu^2/\lambda}$ .

so that the condensate is reduced to its vacuum value when the proper acceleration  $\alpha = 1/\rho$  is turned off. In the opposite limit, one might have been tempted to impose  $\bar{\phi}(\rho \rightarrow 0) = 0$ , leading to a picture of symmetry restoration induced by high acceleration. It is obvious, however, that such a boundary condition is incompatible, which is understood from Eq. (32) that cannot increase the energy of the particle. One might then think that a boundary condition such as  $(d\bar{\phi}/d\rho)_{\rho=0} > 0$  could work to lead to a consistent solution. However, we can easily check by linearizing Eq. (31) around  $\bar{\phi} = 0$  to find that the resulting trajectories are

$$\bar{\phi}(\rho) = C_1 \rho J_{\sqrt{1+\nu^2}}(\mu\rho) + C_2 \rho N_{\sqrt{1+\nu^2}}(\mu\rho), \quad (34)$$

which have zero gradient at  $\rho = 0$ . Therefore, using the boundary condition like  $(d\bar{\phi}/d\rho)_{\rho=0} > 0$  would end up with an inconsistency.

### B. Acceleration catalysis

For this problem, it would be a more reasonable choice to impose one more boundary condition at  $\rho \rightarrow \infty$ . Besides Eq. (33) we also require,

$$\left(\frac{d\bar{\phi}}{d\rho}\right)_{\rho \rightarrow \infty} = 0, \quad (35)$$

so that in the limit of zero acceleration the solution smoothly approaches the value of the condensate in the Minkowski vacuum.

It is straightforward to see that Eq. (31) with  $\mu = 0$  accommodates a “scaling” type of the solution:

$$\bar{\phi}(\rho) = \sqrt{\frac{1+\nu^2}{\lambda}} \frac{1}{\rho}, \quad (36)$$

as recognized first by Stephens [44]. In fact, the scaling solution satisfied the boundary conditions (33) and (35) and is expected to be similar to the full answer with  $\mu^2 > 0$ . In particular the scaling solution suggests that the condensate may blow up as the acceleration increases.

With the conditions (33) and (35) we can solve the equation of motion (32) numerically and we show our numerical results in Fig. 2 for  $\lambda = 0.1$  and  $\lambda = 0.01$ . We find that the condensate grows as  $\rho$  decreases or as the proper acceleration  $\alpha = 1/\rho$  increases. Within the numerical accuracy as  $\rho \rightarrow 0$ , the condensate exhibits diverging behavior similarly to the scaling solution. This increasing behavior of the condensate with acceleration reminds us of the enhancement of the chiral condensate induced by the external magnetic field, which is sometimes referred to as the magnetic catalysis. So, it should be appropriate to name the acceleration-induced enhancement of the condensate *acceleration catalysis*.

## VI. DISCUSSION AND CONCLUSION

Motivated by some discrepancies between the preceding results on the condensate in an accelerated vacuum, we have revisited a real scalar field theory in Rindler spacetime with a spontaneously broken  $Z_2$  symmetry. A key quantity for discussing a possible impact of the acceleration on condensates is the Wightman two-point function in the coincidence limit that represents quantum fluctuations.

First, we have studied the effect of acceleration on the two-point Wightman functions for a free scalar field theory and clarified the meaning of the choices of the vacuum and the observer. This gives a natural explanation for the observation that temperature-like corrections in Rindler spacetime have a sign opposite to the genuine thermal effect. We have argued that such relations of the Wightman function follow from the correspondence to thermo-field dynamics.

The most important part of this paper is the behavior of the scalar condensate as a function of acceleration. Based on our analysis we can conclude that the condensate will not change as long as the system is not accelerated regardless of where the observer sits, that is, the condensate takes the vacuum value in agreement with Ref. [19–21]. This is non-trivial in view of the fact that the Rindler observer perceives thermal effects in the Minkowski vacuum regarding the particle distribution that involves the energy dispersion relation.

Our analysis is, in principle, applied to such a system like a superconductor placed on a transport craft with constant acceleration. If a co-accelerated observer measures a condensate in this superconductor, this observer should see that the condensate changes depending on the acceleration. What we found implies that the scalar condensate increases with increasing acceleration. We have named this phenomenon *acceleration catalysis*.

We would stress that our main result differs from what

is speculated in some papers [22–27]. Ultimately, the fate of the condensate depends on the definition of the coincidence limit of the quantum fluctuation  $\langle \hat{\phi}^2 \rangle$  and the regularization schemes. Our assumption is that a finite deviation in  $\langle \hat{\phi}^2 \rangle$  associated with acceleration should be obtained by subtracting the common divergent pieces. We would also point out that we can in principle judge which of increasing and decreasing scenarios should be the case using a Monte-Carlo simulation on the lattice with non-trivial metric, and a preliminary result favors our scenario of increasing condensate with larger acceleration [47].

Although the above-mentioned subtraction procedure for acceleration physics seems to be generally accepted, there are notable exceptions. Dowker [48] advocated that the Minkowski vacuum fluctuations should be regularized such that the Rindler vacuum fluctuations are subtracted. In our prescription this would correspond to a co-accelerated observer making measurements while taking the Rindler vacuum as a reference point. This situation could be a natural setting for the case of a black hole. It is well-known that Rindler spacetime is an approximation to Schwarzschild spacetime in the near-horizon region. Then, an observer at a fixed distance from the horizon would find a positive thermal-like effect, provided that the observer measures the condensate in the Minkowski vacuum (corresponding to a freely falling frame). Under such conditions it seems conceivable that a condensate would melt as we approach the black-hole horizon [36, 49]. When it comes to acceleration as opposed to well-established thermal physics, the richness of physical outcomes from acceleration deserves further attention.

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## Appendix A: Irrelevance of the observer

Here we articulate a step-by-step demonstration of independence of the field expectation value regardless of the choice of the observer.

Let us first consider the situation with the Minkowski vacuum  $|M\rangle$ . Using the expanded form (14) we find,

$$\begin{aligned} \langle M | \hat{\phi}(x) \hat{\phi}(x') | M \rangle &= \int_0^\infty d\omega \int_0^\infty d\omega' \int d^2 k_\perp \int d^2 k'_\perp \left( f_R f'_R \langle M | \hat{a}_R \hat{a}'_R | M \rangle \right. \\ &\quad + f_R f'_R \langle M | \hat{a}_R \hat{a}'_R | M \rangle + f_R^* f'_R \langle M | \hat{a}_R^\dagger \hat{a}'_R | M \rangle \\ &\quad \left. + f_R^* f'_R \langle M | \hat{a}_R^\dagger \hat{a}'_R | M \rangle \right), \end{aligned} \quad (\text{A1})$$

where we used the expression for  $\hat{\phi}$  quantized in Rindler spacetime and we introduced a compact notation with the prime for quantities with  $\omega'$  and  $\mathbf{k}'_\perp$ . We plug Eq. (18) and similar expressions for other combinations of the creation/annihilation operators into Eq. (A1). We further replace  $\omega$  with  $-\omega$  to obtain:

$$\begin{aligned} \langle M | \hat{\phi}(x) \hat{\phi}(x') | M \rangle &= \frac{1}{2(2\pi)^4} \int \frac{dk^+}{2\pi k^+} \int d^2 k_\perp e^{i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} \\ &\times \left[ \int_0^\infty d\omega F(\omega, k^+, \mathbf{k}_\perp) \int_0^\infty d\omega' G(\omega', k^+, \mathbf{k}_\perp) \right. \\ &\quad \left. + \int_{-\infty}^0 d\omega F(\omega, k^+, \mathbf{k}_\perp) \int_{-\infty}^0 d\omega' G(\omega', k^+, \mathbf{k}_\perp) \right], \end{aligned} \quad (\text{A2})$$

with the following functions that we define by

$$F(\omega, k^+, \mathbf{k}_\perp) = e^{-i\omega\eta} e^{i\omega \log(k^+ \sqrt{2}/\kappa)} K\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right), \quad (\text{A3})$$

$$G(\omega', k^+, \mathbf{k}_\perp) = e^{i\omega'\eta'} e^{-i\omega' \log(k^+ \sqrt{2}/\kappa)} K\left(\omega', \frac{\kappa\rho'}{2}, \frac{\kappa\rho'}{2}\right). \quad (\text{A4})$$

These are, of course, functions of  $t$  and  $z$  through  $\eta$  and  $\rho$ , which is not indicated in the argument of  $F$  and  $G$  for notational brevity. Also, in deriving Eq. (A2), we employed the analyticity property of the function  $K(\omega, \alpha, \beta)$ . Now we can deform the quantity in the angle parentheses of Eq. (A2) as

$$\begin{aligned} &\int_0^\infty d\omega F \int_0^\infty d\omega' G + \int_{-\infty}^0 d\omega F \int_{-\infty}^0 d\omega' G \\ &\quad + \int_0^\infty d\omega F \int_{-\infty}^0 d\omega' G + \int_{-\infty}^0 d\omega F \int_0^\infty d\omega' G \quad (\text{A5}) \\ &= \int_{-\infty}^\infty d\omega F \int_{-\infty}^\infty d\omega' G, \end{aligned}$$

without changing its value; two latter terms among four in the left-hand side of the above expression give no contribution under the  $k^+$  integration. After the  $k^+$  integration, in fact, we can easily show that the resulting contribution is proportional to  $\delta(\omega - \omega')$ . It is obvious that such a singularity at  $\omega = \omega'$  in Dirac's delta function cannot be picked up in the integrals like  $\int_0^\infty d\omega \int_{-\infty}^0 d\omega'$  and  $\int_{-\infty}^0 d\omega \int_0^\infty d\omega'$ .

Eventually, we can rewrite the two-point function into



the standard form in Minkowski spacetime as

$$\begin{aligned}
& \langle M | \hat{\phi}(x) \hat{\phi}(x') | M \rangle \\
&= \frac{1}{2(2\pi)^4} \int \frac{dk^+}{2\pi k^+} \int d^2 k_\perp e^{i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} \\
&\quad \times \int_{-\infty}^{\infty} d\omega F(\omega, k^+, \mathbf{k}_\perp) \int_{-\infty}^{\infty} d\omega' G(\omega', k^+, \mathbf{k}_\perp) \\
&= \int \frac{dk^+}{2k^+} \int d^2 k_\perp e^{i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{ik^+(x^- - x'^-)} e^{ik^-(x^+ - x'^+)} .
\end{aligned} \tag{A6}$$

In this concrete process of calculations we note that we used Eq. (19) from the second line to the third line of Eq. (A6).

Next, we consider the situation with the Rindler vacuum  $|R\rangle$ . We can find,

$$\begin{aligned}
& \langle R | \hat{\phi}(x) \hat{\phi}(x') | R \rangle \\
&= \frac{1}{4\pi^4} \int_0^\infty d\omega \int d^2 k_\perp e^{i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i\omega(\eta - \eta')} \\
&\quad \times \sinh(\pi\omega) K_{i\omega}(\kappa\rho) K_{i\omega}(\kappa\rho')
\end{aligned} \tag{A7}$$

using the field operator in terms of the Rindler basis functions. On the other hand, we have,

$$\begin{aligned}
& \langle R | \hat{\phi}(x) \hat{\phi}(x') | R \rangle = \int_0^\infty dk^+ \int_0^\infty dk^{+'} \int d^2 k_\perp \int d^2 k'_\perp \\
&\quad \times \left( f f' \langle R | \hat{a}_1 \hat{a}'_1 | R \rangle + f f'^* \langle R | \hat{a}_1 \hat{a}'_1{}^\dagger | R \rangle \right. \\
&\quad \left. + f^* f' \langle R | \hat{a}_1^\dagger \hat{a}_1 | R \rangle + f^* f'^* \langle R | \hat{a}_1^\dagger \hat{a}'_1{}^\dagger | R \rangle \right) ,
\end{aligned} \tag{A8}$$

where we introduced a compact notation;  $f'$  and  $\hat{a}'_1$  for quantities with  $x'$ . Using the definition (15) and the Bogolyubov transformation (16) we have,

$$\begin{aligned}
& \langle R | \hat{a}_1(\mathbf{k}_\perp, k^+) \hat{a}_1(\mathbf{k}'_\perp, k'^+) | R \rangle \\
&= -\delta^{(2)}(\mathbf{k}_\perp + \mathbf{k}'_\perp) \frac{1}{k^+ k'^+} \\
&\quad \times \int_0^\infty \frac{d\omega}{2\pi} \alpha_\omega \beta_\omega e^{-i\omega \log(k^+ \sqrt{2}/\kappa)} e^{i\omega \log(k'^+ \sqrt{2}/\kappa')} ,
\end{aligned} \tag{A9}$$

and similar expressions for the expectation values involving other combinations of the annihilation and creation operators. Now we plug these terms into Eq. (A8) and use Eq. (12) to define the  $K$  function in a form of the  $k^+$

and  $k^{+'}$  integrations. For example, we can show:

$$\begin{aligned}
& \int_0^\infty \frac{dk^+}{k^+} e^{ik^+ x^- + ik'^+ x'^-} e^{-i\omega \log(k^+ \sqrt{2}/\kappa)} \\
&= K\left(\omega, \frac{\kappa\rho}{2} e^\eta, \frac{\kappa\rho}{2} e^{-\eta}\right) .
\end{aligned} \tag{A10}$$

Therefore, only the integrations over  $\mathbf{k}_\perp$  and  $\omega$  remain. Collecting all four terms, we finally find,

$$\begin{aligned}
& \langle R | \hat{\phi}(x) \hat{\phi}(x') | R \rangle \\
&= \frac{1}{2(2\pi)^4} \int_0^\infty d\omega \int d^2 k_\perp e^{i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i\omega(\eta - \eta')} \\
&\quad \times (\alpha_\omega^2 - 2\alpha_\omega \beta_\omega e^{-\pi\omega} + \beta_\omega^2 e^{-2\pi\omega}) \\
&\quad \times K\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) K\left(\omega, \frac{\kappa\rho'}{2}, \frac{\kappa\rho'}{2}\right) .
\end{aligned} \tag{A11}$$

Using  $\alpha_\omega^2 - 2\alpha_\omega \beta_\omega e^{-\pi\omega} + \beta_\omega^2 e^{-2\pi\omega} = 1 - e^{-2\pi\omega}$  together with Eq. (13) we easily see that Eq. (A11) is reduced to Eq. (A7).

Now, finally, we want to sketch how this two-point function result can be generalized for  $n$ -point ones. We must understand that  $\hat{\phi}$  quantized in the Rindler vacuum is simply a restriction of  $\hat{\phi}$  on the Rindler right-wedge. We can explicitly see this from expressions of the type like Eq. (19).

Starting with the Minkowski quantized  $\hat{\phi}$ , therefore, we have,

$$\begin{aligned}
& \hat{\phi} = \int_0^\infty d\omega \int \frac{d^2 k_\perp}{[2(2\pi)^4]^{1/2}} \left[ \hat{a}_2(\mathbf{k}_\perp, \omega) K\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) \right. \\
&\quad \left. + \hat{a}_2^\dagger(-\mathbf{k}_\perp, -\omega) K^*\left(-\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) \right] e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp - i\omega\tau} \\
&\quad + \int_0^\infty d\omega \int \frac{d^2 k_\perp}{[2(2\pi)^4]^{1/2}} \left[ \hat{a}_2^\dagger(\mathbf{k}_\perp, \omega) K^*\left(\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) \right. \\
&\quad \left. + \hat{a}_2(-\mathbf{k}_\perp, -\omega) K\left(-\omega, \frac{\kappa\rho}{2}, \frac{\kappa\rho}{2}\right) \right] e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp + i\omega\tau} .
\end{aligned} \tag{A12}$$

By using the Bogolyubov transformations (16), the restriction to the right-wedge ( $\rho > 0$ ) gives  $\hat{\phi}_R$ , while the restriction to the left-wedge ( $-\rho > 0$ ) gives  $\hat{\phi}_L$ . Overall, we can write:

$$\hat{\phi} = \theta(\rho) \hat{\phi}_R + \theta(-\rho) \hat{\phi}_L . \tag{A13}$$

Since the annihilation and creation operators from the right-wedge and left-wedge mutually commute, and since left-wedge operators do not act on  $|R\rangle$  by definition, it follows that

$$\begin{aligned}
& \langle R | \hat{\phi}(x_1) \hat{\phi}(x_2) \dots \hat{\phi}(x_n) | R \rangle \\
&= \langle R | \hat{\phi}_R(x_1) \hat{\phi}_R(x_2) \dots \hat{\phi}_R(x_n) | R \rangle .
\end{aligned} \tag{A14}$$

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